

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/309967859>

Imagined Speech Classification using EEG

Article in *Advances in Biomedical Science and Engineering* · December 2014

CITATIONS

22

READS

2,498

4 authors, including:



Kamalakkannan Ravi

University of Central Florida

9 PUBLICATIONS 32 CITATIONS

SEE PROFILE

Imagined Speech Classification using EEG

Kamalakkannan R. *, Rajkumar R., Madan Raj. M., Shenbaga Devi. S.

Department of Electronics and Communication Engineering, College of Engineering, Guindy Campus, Anna University, Chennai - 600 025, India.

*Corresponding author: rkk.bme@gmail.com

Abstract:

The objective of this work is to assess the possibility of using (Electroencephalogram) EEG for communication between different subjects. Here EEG signals are recorded from 13 subjects by inducing the subjects to imagine the English vowels 'a', 'e', 'i', 'o' and 'u' through visual stimulus. These recorded signals are then processed to remove artifacts and noise. Common features: Average power, Mean, Variance and Standard deviation are computed and classified using bipolar neural network. This method yields maximum classification accuracy of 44%. The result shows that EEG has some distinctive information for across subject classification.

Keywords:

EEG; Imagined Speech; Classification; Bipolar Neural Network; Brain Computer Interface

1. INTRODUCTION

Early work by Wester, M. [1] showed that unspoken speech produced particular brain activities. However, studies hypothesized that the results are overestimated due to temporal correlated artifacts. It is then proved by Porbadnigk *et al* [2] that proved unspoken speech can be recognized using methods used by Wester.

Meanwhile D'Zmura *et al* [3] have found that imagination of syllable /ba/ and /ku/ has effect on EEG alpha, beta and theta bands. And also Brigham, K. and Kumar, B.V.KV. classified the same syllables for silent communication [4] and authentication [5]. They have used autoregressive model along with Hurst exponent to classify imagined speech syllables with a good degree of accuracy.

Similarly Chi *et al* [6] classified 5 imagined phonemes and also discriminated these signals from those generated during periods of no imagined speech. Naive Bayesian linear discriminant analysis classification methods were applied to EEG signals that were recorded during imagined phoneme production. Recently, Matsumoto, M and Hori, J [7] classified Event-related potentials (ERPs) obtained from imagined vocalization of Japanese vowels where the classification accuracy was significantly improved while using the adaptive collection.

Our work involves recording the EEG waveform from a subject while he/she is visualizing the alphabet characters 'a', 'e', 'i', 'o' and 'u'. These five letters are chosen such that they convey maximum variation in vocal articulation [8] and hypothesized to show similar variance in EEG waveform. EEG Signals are preprocessed for feature extraction and classified using Back Propagation Neural Network.

The following section describes the methodology explains data collection, preprocessing, feature extraction and classification and Section III discusses the results.

2. METHODOLOGY

For this study, 13 volunteered subjects' (10 male, 3 female) EEG are recorded and the average age is 21 years.

2.1 Data Collection

1. Subject Preparation

In order to reduce electrode-scalp interface impedance, subject scalp is cleaned with skin preparation gel. Then the electrode is placed on the scalp using EEG paste. Electrodes are placed as per International 10–20 system. **Figure 1(a)** shows side view and **Figure 1(b)** shows top view of International 10 – 20 electrode placement system.

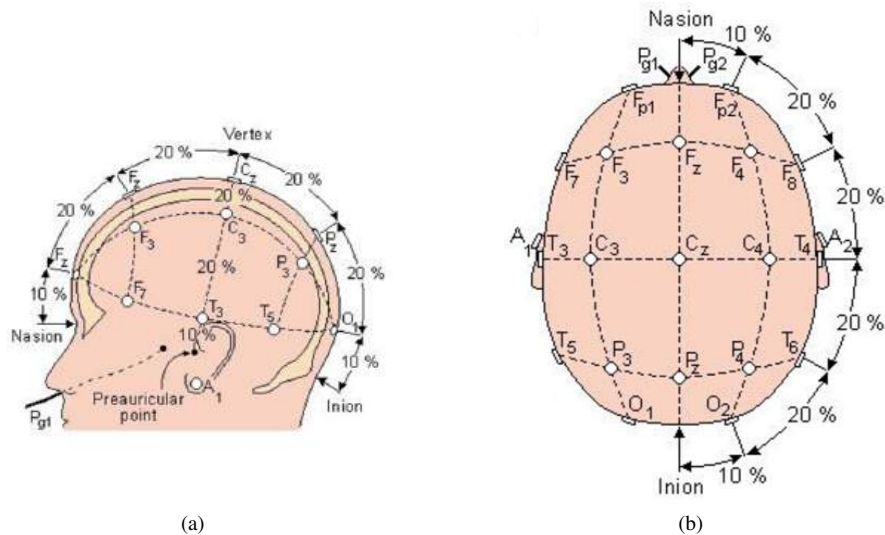


Figure 1. International 10 – 20 system of Electrode placement as standardized by the American Electroencephalographic Society. (Redrawn from Sharbrough, 1991).

2. Stimulus Preparation

To ensure that the subject is visualizing the desired vowel, visual stimulus is given. The visual stimuli are English language vowels 'a', 'e', 'i', 'o' and 'u'. At the starting of trial, 6 second is given for relaxation and for 2 seconds a vowel is displayed and 2 second relaxation time is given and likewise 6 times a vowel is shown in a single trial of 30 seconds. So every single trial has 6 stimuli instances. **Figure 2** shows the stimulus flow.

3. EEG Recording Method

EEG data from the subject is recorded using EEG machine in monopolar configuration with 256Hz

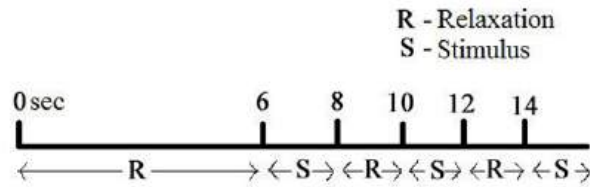


Figure 2. Stimulus flow for a vowel.

sampling frequency.

Subject is seated in a faintly lit room. Then electrodes are placed on the scalp and electrode - scalp impedance is kept below 20K Ohm. The subject is instructed to be in a relaxed condition with his/her eyes closed and also instructed to avoid any eye blinks, tongue movement and muscle movement during recording. The subject opens his/her eyes; visual stimulus and recording are started simultaneously. Recording is stopped at the end of visual stimulus and the subject closes his/her eyes for relaxation. Likewise 5 trials are taken for each vowel that results in 25 trials per subject. Thus, totally 325 trials are taken for 13 different subjects.

The EEG signal is recorded on all the 20 electrodes specified by the 10-20 system. The recorded EEG is exported and the data is preprocessed. Features are then extracted and finally classification of the data is done which are explained in the next sections. Figure 3 shows 16 channel EEG wave form recorded for 10 seconds.

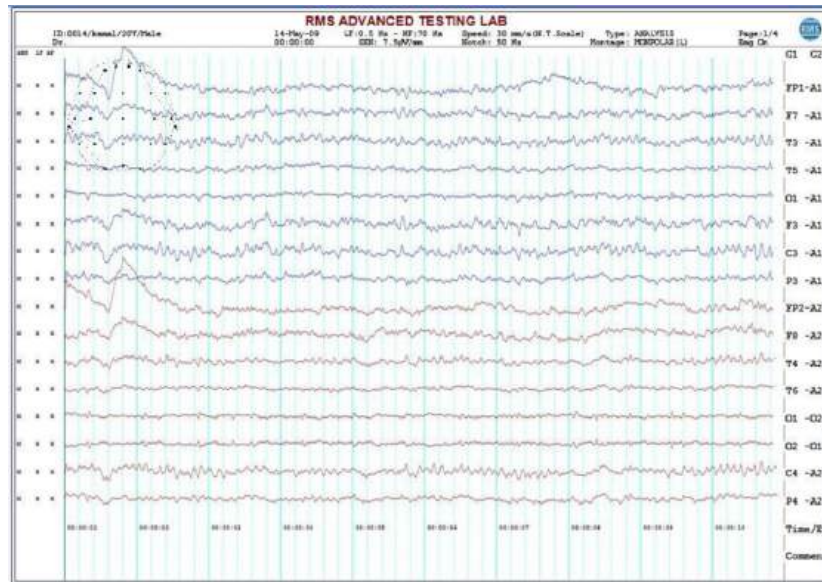


Figure 3. 10 Sec - Recorded EEG comprising 6 sec relaxation, 2 sec stimulation and 2 sec relaxation.

2.2 Preprocessing

EEG signals are prone to Electromyogram (EMG) artifacts, Electrooculogram (EOG) artifacts, baseline wandering, powerline interference and eye blink artifacts. EMG artifacts are removed by using inbuilt filter in the EEG machine. Subjects are instructed to avoid eye blinking to eliminate eye blink artifacts.

Using 2nd order IIR notch filter, powerline interference is removed [9]. Baseline wandering is removed using wavelet decomposition [10] method with eight levels and the eighth level approximation coefficients are made zero.

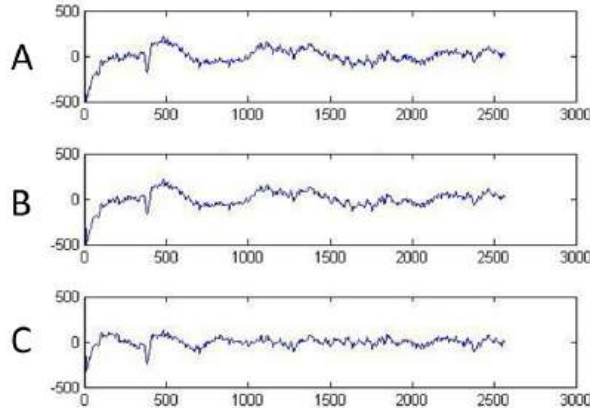


Figure 4. Preprocessed signals.

These signals are further processed to obtain the relevant features which are explained in the next section.

2.3 Feature Extraction

This section details the different features extracted from the preprocessed signal and feature justification. Any signal is characterized by its mean value, variance and average power and these features are considered here for analysis.

1. Estimation of Mean

The statistical parameter, mean indicated by μ is the average value of a signal. Mathematical form is:

$$\mu = \frac{1}{N} \sum_{n=0}^{N-1} x_n$$

2. Estimation of Variance and Standard Deviation

Variance measures how far a set of numbers is spread out. Variance formula is

$$\sigma^2 = \frac{1}{N} \sum_{n=0}^{N-1} (x_n - \mu)^2$$

The standard deviation is the average deviation; the averaging is done with signal amplitude. This is

achieved by squaring each of the deviations before taking the average. In equation form, the standard deviation is calculated:

$$\sigma = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} (x_n - \mu)^2}$$

3. Estimation of Average Power

To compute the average power, Fast Fourier transform (FFT) is computed from the signal. A Fast Fourier transform (FFT) is an algorithm to compute more quickly, the Discrete Fourier Transform (DFT) and its inverse. The DFT is defined by the formula

$$X(f) = \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}$$

Then Power Spectral Density (PSD) is computed using FFT. The PSD is the average of the Fourier transform magnitude squared, over a large time interval which is defined by the formula

$$S(f) = |X(f)|^2$$

Finally from the PSD, average power is found Its mathematical form is:

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} S(f)$$

Features computed from the preprocessed signals are its average power, Mean, Variance and Standard deviation. Features are computed for both relaxation and stimulation instances which are shown in **Table 1** and **Table 2**. The features during stimulation period discriminate each vowel evidently. Thus it shows difference among both intra and inter subject imagined vowels. As the feature values during relaxation period vary for each subject, the difference between the relaxation and stimulation values are computed which is shown in **Table 3**. This difference values are taken as the final features for classification. The final (difference) features are used to differentiate 5 vowels for 13 subjects which are shown in **Figure 5** to **Figure 17**.

Table 1. Stimulation period feature values, Here APWR – Average power (Watt), VAR – Variance, SD – Standard deviation.

SUBJECT		1	2	3	4	5	6	7	8	9	10	11	12	13
LETTERS	PARAMETER													
a	APWR	246.1806	146.5056	2210.583	146.5056	1321.74	201.1038	1967.358	1916.348	2157.295	4281.892	2264.866	5770.546	2045.86
	MEAN	12.40849	0.111319	0.575345	0.111319	0.101759	32.99338	0.149963	0.216042	0.302178	2.872673	0.927232	-1.24229	0.61654
	VAR	469.9594	292.254	4375.406	292.254	2623.581	401.2625	3905.23	3776.094	4264.11	8187.644	4486.754	11393.31	4092.351
	SD	20.26729	16.1568	45.61844	16.1568	32.86768	17.25009	37.12774	44.53899	53.58178	56.56957	24.88666	47.10334	46.2024
e	APWR	463.1008	164.0027	2136.47	630.6664	1725.812	1075.723	1576.211	1027.651	2232.969	4196.269	386.6323	2705.76	7082.946
	MEAN	0.040335	-0.12646	-0.2873	-0.15104	0.202295	12.17738	0.172982	-0.38464	2.207803	0.165224	-0.11094	-0.30744	202.6203
	VAR	918.8943	326.9315	4234.765	1258.116	3439.565	2145.766	3127.936	2028.514	5439.902	8323.427	766.7931	5302.256	13173.69
	SD	25.2336	17.278	47.1836	25.74224	32.87896	37.6884	36.96596	31.94939	78.82632	57.20074	14.69795	47.4772	482.7414
i	APWR	984.4371	127.7354	6867.443	127.7354	1143.434	1487.866	1438.992	1447.391	3015.677	3882.879	454.4454	3916.779	4752.508
	MEAN	-0.46753	-0.08207	1.019354	-0.08207	0.28478	0.315436	0.015922	-0.03608	-0.56919	0.712443	-0.2633	1931.971	68.25735
	VAR	1971.422	254.7648	13593.94	254.7648	2258.941	2926.517	2852.469	2856.629	5925.516	7697.006	826.5625	5839.969	8373.054
	SD	28.71564	15.04424	71.72487	15.04424	33.60665	39.08048	33.49779	40.98327	51.65866	56.27883	109.3687	53.34694	48.49589
o	APWR	2049.049	131.929	8464.454	652.1769	868.9646	927.8206	1887.433	2250.3	3037.938	2743.246	242.6185	10845.77	1671.933
	MEAN	6.755405	0.062203	119.4416	0.17762	0.041175	0.873436	-0.1133	-0.9289	0.012516	1.316021	0.262586	0.792916	-0.81444
	VAR	4045.149	263.371	16698.73	1302.15	1720.685	1834.063	3750.029	4428.019	5977.956	5427.706	480.491	21335.54	3315.733
	SD	36.17089	15.3698	220.3983	25.64937	29.22299	32.96869	36.77524	51.52037	56.31488	50.28649	29.84794	100.1371	27.77397
u	APWR	734.9513	168.1696	1846.312	438.89	1086.049	628.7174	1381.233	1780.789	4817.327	4608.574	449.3396	2157.295	1178.252
	MEAN	0.396842	-0.01841	0.69811	0.01251	0.152183	0.054878	0.012297	-0.19655	2.452512	158.0317	-0.02786	0.302178	0.438606
	VAR	1457.589	335.6202	3667.372	877.4173	2155.299	1237.969	2748.806	3515.204	9340.154	9124.248	854.1672	4264.11	2334.678
	SD	28.05244	27.86449	45.2046	20.74335	31.33168	27.75394	32.28254	45.06093	71.52988	61.43686	50.68344	53.58178	17.40512

Table 2. Relaxation period feature values.

SUBJECT		1	2	3	4	5	6	7	8	9	10	11	12	13
LETTERS	PARAMETER													
a	APWR	245.2773	142.5598	2257.636	473.3102	1207.334	178.2763	2394.731	2023.799	2074.371	4272.149	1852.971	1796.095	2073.679
	MEAN	-0.18455	0.078125	0.787039	14.17743	-0.27702	0.4563	0.165213	98.37077	-0.75551	-0.08466	0.034722	1207.085	1.25542
	VAR	488.9652	284.3667	4282.997	944.8458	2396.709	349.6402	4710.556	3990.25	4095.129	8469.229	3662.538	1944.572	3867.003
	SD	20.1578	15.94731	45.07018	24.46399	31.28875	16.50629	40.51532	45.61631	52.18878	58.40423	23.11143	62.11882	44.61854
e	APWR	474.5867	129.1502	2183.271	475.1335	1634.78	1085.885	1714.315	1091.793	2458.276	4180.423	412.3623	3185.688	6949.867
	MEAN	0.025809	-0.11014	-0.18898	-0.0621	-0.08588	0.647802	0.557686	-0.01435	1.324017	0.334959	-0.14465	304.2736	1.635018
	VAR	941.674	312.6975	4326.631	947.9454	3258.12	2142.81	3400.111	2149.857	5981.458	8292.635	813.1506	5935.217	13825.68
	SD	25.4494	24.97777	47.04577	20.63822	31.61453	45.67854	37.79678	32.9272	48.12964	57.09591	14.95535	51.83343	91.68945
i	APWR	1004.469	113.3671	7043.129	481.4588	1163.268	2132.891	1235.638	1475.829	3461.775	4060.414	390.845	3697.625	4515.831
	MEAN	-0.40172	-0.1204	0.937585	0.037054	0.042963	0.34567	0.350296	-0.6124	0.26467	726.8692	-0.25044	1.307905	827.3785
	VAR	1992.612	219.8511	13944.06	962.3798	192291.75	2806.232	2431.119	2914.896	6848.408	8058.825	774.6292	7289.527	8909.458
	SD	29.06454	11.98199	72.81805	22.97187	33.46558	14.56361	31.34133	40.80613	54.71988	57.48004	14.13217	50.25456	49.53325
o	APWR	2275.166	110.8167	11002	760.67	758.5783	1096.133	2182.464	2193.985	3051.694	2860.932	262.0431	9191.61	13210.86
	MEAN	0.420614	-0.01093	0.574326	0.108218	0.073446	-0.24498	0.125019	-0.49997	-0.63024	-0.05882	-0.14496	0.256619	-4.40901
	VAR	4473.221	206.3348	16058.24	1519.246	1500.644	2165.434	4344.027	4317.183	6023.092	5664.169	519.4063	18997.56	26211.73
	SD	37.12226	12.3335	87.10614	27.30441	27.64973	38.93964	39.15829	51.2277	56.37124	51.48688	12.7012	95.09584	209.2286
u	APWR	714.6756	167.5592	1823.513	513.8567	1021.641	1252.247	1336.74	1779.753	5146.436	4527.838	435.1683	3917.638	10099.12
	MEAN	0.311113	4.363272	40.9318	-0.02248	0.228115	0.2345	0.183497	0.104008	2.058331	366.2253	0.033114	0.76227	0.115991
	VAR	1417.001	323.4276	3620.751	1027.395	2026.669	2464.033	2632.174	3509.863	10073.51	8958.748	865.9917	7713.523	20038.18
	SD	27.63833	26.59622	128.6013	21.76294	30.77094	55.10202	31.88064	44.29075	73.53618	61.12957	15.09209	47.957	171.9503

Table 3. Difference values between features in relaxation period and stimulus period.

SUBJECT		1	2	3	4	5	6	7	8	9	10	11	12	13
LETTER	PARAMETER													
a	APWR	0.903333	-326.805	-47.0531	114.4056	22.8275	-107.451	9.743229	82.92375	3.945833	-427.373	411.8942	5674.344	-27.8194
	MEAN	12.59305	-14.0661	-0.21169	0.378781	32.53708	-98.1547	2.957336	1.057684	0.033194	-0.01525	0.89251	-11.1966	-0.63888
	VAR	-19.0058	-652.592	92.40958	226.8719	51.62226	-214.156	-281.585	168.9813	7.887292	-805.325	824.2167	11225.68	225.3483
	SD	0.109483	-8.30719	0.548262	1.578927	0.743798	-1.07732	-1.83466	1.392991	0.20949	-3.38758	1.775236	50.64541	1.583861
e	APWR	-11.4859	155.5329	-46.8013	91.03146	-10.1619	-64.1417	15.84667	-225.306	34.8525	-138.104	-25.73	-479.927	133.0787
	MEAN	0.014527	-0.08895	-0.09832	0.288173	11.52958	-0.37029	-0.16973	0.883786	-0.01631	-0.3847	0.033709	-304.581	200.9853
	VAR	-22.7797	310.1705	-91.8667	181.4446	2.95625	-121.343	30.79167	-541.556	14.23396	-272.175	-46.3575	-632.96	-651.985
	SD	-0.21579	5.104021	0.137828	1.264429	-7.99014	-0.97781	0.104832	-402.47	-7.69977	-0.83082	-0.2574	-4.35624	391.0519
i	APWR	-20.0317	-353.723	-175.685	-19.8342	-645.025	-28.4383	-177.535	-446.098	14.36831	203.354	63.60042	219.1542	236.6771
	MEAN	-0.06581	-0.11913	0.081769	0.241817	-0.03023	0.57632	-726.157	-0.83386	0.038325	-0.33437	-0.00286	1930.664	-759.121
	VAR	-21.1898	-707.615	-350.119	-1.90E+07	120.2844	-58.2665	-361.819	-922.893	34.91368	421.3508	51.93333	-1449.56	-36.4044
	SD	-0.34889	-7.92763	-1.09318	0.141066	24.51687	0.177142	-1.20121	-3.06122	3.062243	2.15646	95.23657	3.092378	-1.03736
o	APWR	-226.117	-108.493	-2537.55	110.3863	-168.312	56.31458	-117.686	-13.7562	21.11229	-295.031	-19.4246	1654.155	-11538.9
	MEAN	6.334791	0.069402	118.8673	-0.03227	1.118412	-0.42893	1.374841	0.642755	0.07313	-0.23832	0.407544	0.536297	3.594576
	VAR	-428.071	-217.097	640.4848	220.0415	-331.372	110.8354	-236.463	-45.1354	57.03625	-593.998	-38.9152	2337.977	-22896
	SD	-0.95136	-1.65504	133.2922	1.573263	-5.97095	0.292671	-1.20039	-0.05636	3.0363	-2.38304	17.14674	5.041299	-181.455
u	APWR	20.27563	-74.9667	22.79854	64.40854	-623.53	1.035834	80.73583	-329.109	0.610375	44.49313	14.17125	-1760.34	-8920.87
	MEAN	0.085729	0.034991	-40.2337	-0.07593	-0.17962	-0.30056	-208.194	0.394181	-4.38168	-0.1712	-0.06097	-0.46009	0.322616
	VAR	40.58813	-149.978	46.62104	128.6302	-1226.06	5.341667	165.5	-733.354	12.19265	116.6313	-11.8245	-3449.41	-17703.5
	SD	0.414109	-1.01959	-83.3967	0.560741	-27.3481	0.770185	0.307285	-2.0063	1.268264	0.4019	35.59135	5.624772	-154.545

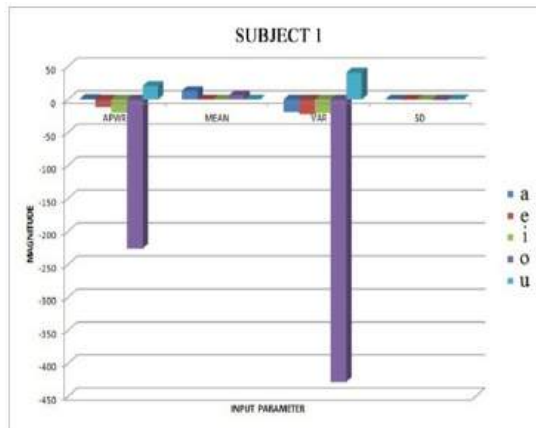


Figure 5. Vowels discrimination for subject 1 using difference values.



Figure 6. Vowels discrimination for subject 2 using difference values.

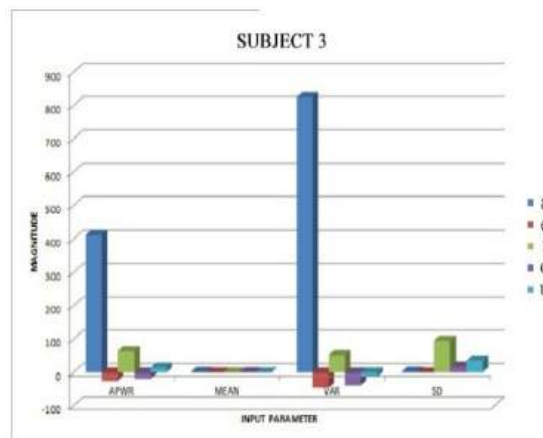


Figure 7. Vowels discrimination for subject 3 using difference values.

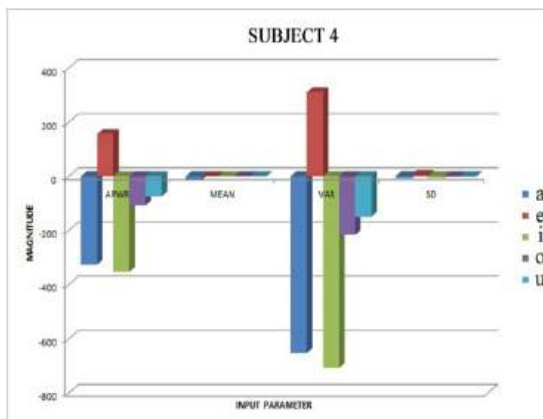


Figure 8. Vowels discrimination for subject 4 using difference values.

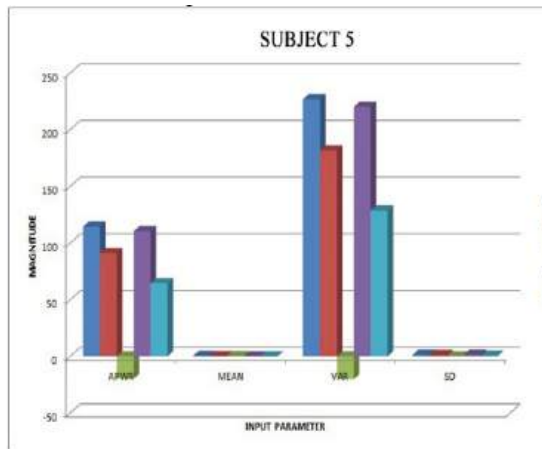


Figure 9. Vowels discrimination for subject 5 using difference values.

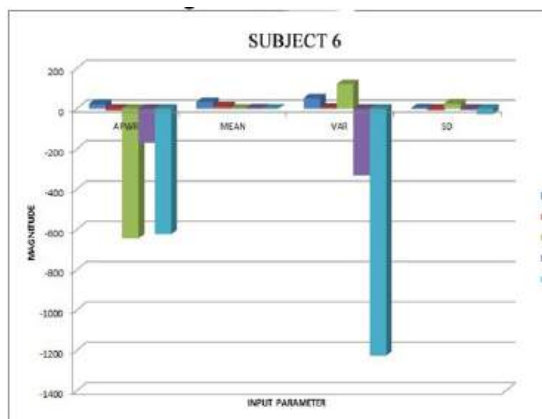


Figure 10. Vowels discrimination for subject 6 using difference values.

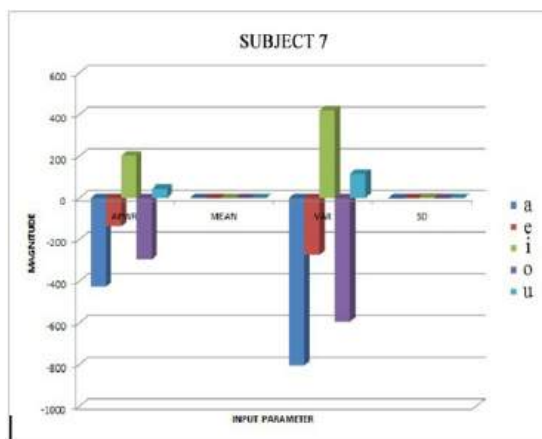


Figure 11. Vowels discrimination for subject 7 using difference values.

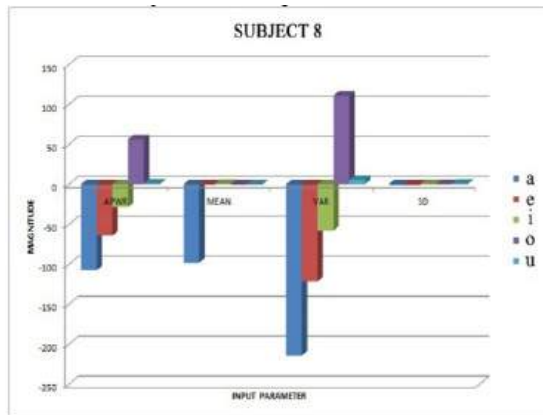


Figure 12. Vowels discrimination for subject 8 using difference values.

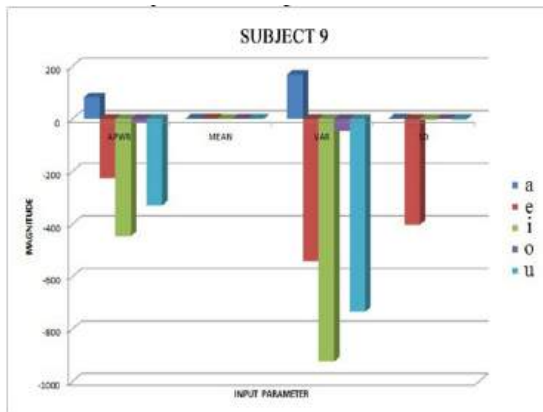


Figure 13. Vowels discrimination for subject 9 using difference values.

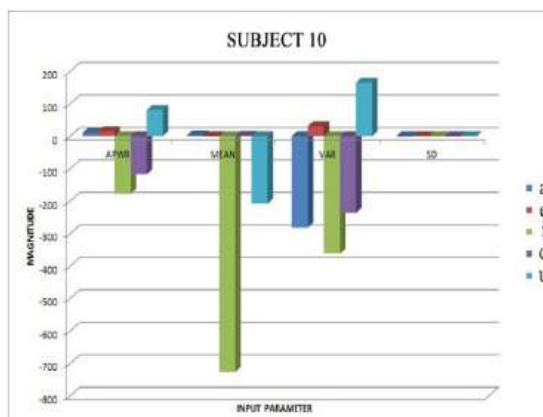


Figure 14. Vowels discrimination for subject 10 using difference values.

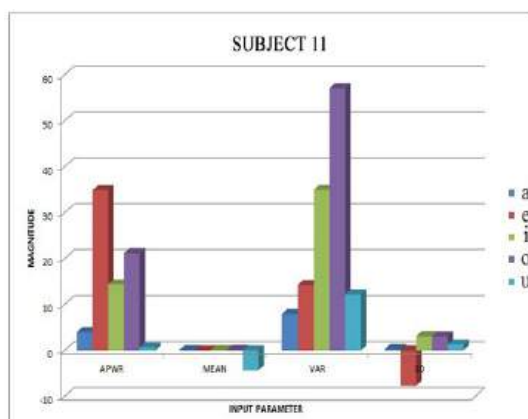


Figure 15. Vowels discrimination for subject 11 using difference values.

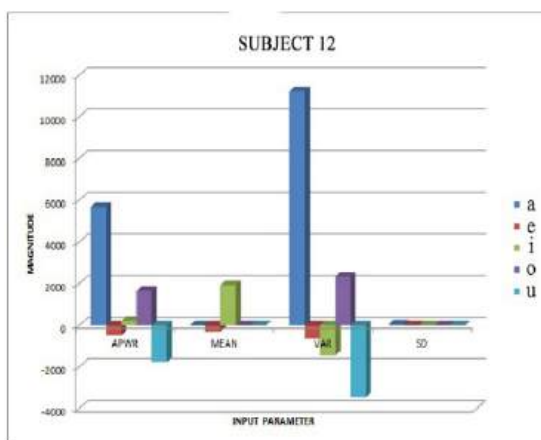


Figure 16. Vowels discrimination for subject 12 using difference values.

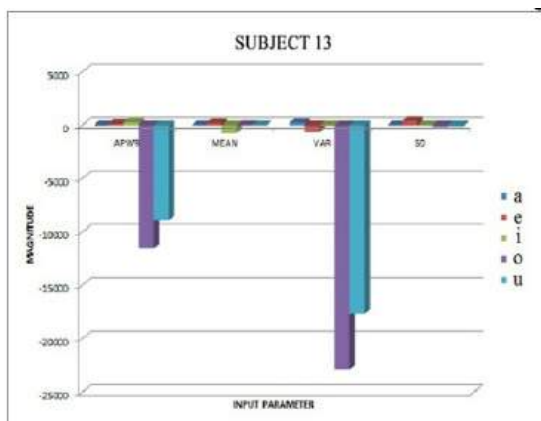


Figure 17. Vowels discrimination for subject 13 using difference values.

These final (difference) features are classified which is explained in next section.

3. CLASSIFICATION

Back Propagation Neural Network is used for classification. It has 2 hidden layers containing 9 and 5 nodes respectively. The input layer consists of 4 nodes corresponding to 4 input parameters namely average power, mean, variance and standard deviation, while the output layer of 5 nodes representing characters 'a', 'e', 'i', 'o' and 'u'. Classification is done using 4 linear features namely average power, mean, variance, standard deviation computed from 13 subjects' EEG signal. Of the total data, 10 subjects' data (~80%) are allocated for training the network and 3 subjects' data (~20%) for testing.

4. RESULTS AND DISCUSSION

Each subject data is classified 20 times (20 trials) and results are tabulated.

Table 4 shows the classification results of each subject with letter wise average, Minimum and Maximum. Vowels 'a', 'i' and 'u' are classified significantly with average classification rates 65%, 40% and 41.6% respectively.

Table 4. Classification results - letter wise.

Subjects	Classification rate (%)				
	'a'	'e'	'i'	'o'	'u'
Subject 1	55	35	75	10	45
Subject 2	80	15	15	15	35
Subject 3	60	5	30	20	45
Letter wise Average	65	18.3	40	15	41.6
Letter wise Min	55	5	15	10	35
Letter wise Max	80	35	75	20	45

Table 5 shows the classification results of each subject with its average, minimum and maximum classification rate. The average classification rates of 3 subjects are 44%, 32% and 32%. Classification rates can be improved by increasing number of features and training set data.

Table 5. Classification results - letter wise.

Subjects	Classification rate (%)					Subject wise Average
	'a'	'e'	'i'	'o'	'u'	
Subject 1	55	35	75	10	45	44
Subject 2	80	15	15	15	35	32
Subject 3	60	5	30	20	45	32

5. CONCLUSION AND FUTURE WORK

The average classification rate is 44% at maximum suggests that here is some distinctive information contained in the EEG data for different imagined speech and our method is able to classify them. Importantly it proves across possibility of subject classification. It is suspected that increase in relaxation

time would increase the classification accuracy. So an experiment must be set up to assess the effect of relaxation time in the classification accuracy.

This could be the first step towards making a device that allows paralyzed patients and those who are unable to talk or to communicate. The number of alphanumeric characters under consideration can also be increased to check if it is feasible to make a device that can decode all 36 major alphanumeric characters. Then there is a possibility of texting by just imagining instead of typing them.

6. CONFLICT OF INTEREST AND ETHICAL CLEARANCE

The authors declare no potential conflicts of interest with respect to the research, authorship, publication of the article and any support from third party is noted. The volunteers are informed about the experiment procedure to make their own decision to whether to participate or not in the experiment. Only interested volunteers are involved in the data acquisition.

References

- [1] M. Wester and T. Schultz, "Unspoken speech-speech recognition based on electroencephalography," *Master's thesis, Universität Karlsruhe (TH), Karlsruhe, Germany*, 2006.
- [2] A. Porbadnigk, M. Wester, and T. S. Jan-p Calliess, "EEG-based speech recognition impact of temporal effects," 2009.
- [3] M. DZmura, S. Deng, T. Lappas, S. Thorpe, and R. Srinivasan, "Toward EEG sensing of imagined speech," in *Human-Computer Interaction. New Trends*, pp. 40–48, Springer, 2009.
- [4] K. Brigham and B. V. Kumar, "Imagined speech classification with EEG signals for silent communication: a preliminary investigation into synthetic telepathy," in *Bioinformatics and Biomedical Engineering (iCBBE), 2010 4th International Conference on*, pp. 1–4, IEEE, 2010.
- [5] K. Brigham and B. V. Kumar, "Subject identification from electroencephalogram (EEG) signals during imagined speech," in *Biometrics: Theory Applications and Systems (BTAS), 2010 Fourth IEEE International Conference on*, pp. 1–8, IEEE, 2010.
- [6] X. Chia, J. B. Hagedorna, D. Schoonovera, and M. D'Zmuraa, "EEG-Based Discrimination of Imagined Speech Phonemes," *International Journal of Bioelectromagnetism*, vol. 13, no. 4, pp. 201–206, 2011.
- [7] M. Matsumoto and J. Hori, "Classification of silent speech using adaptive collection," in *Computational Intelligence in Rehabilitation and Assistive Technologies (CIRAT), 2013 IEEE Symposium on*, pp. 5–12, IEEE, 2013.
- [8] R. A. Mitchell and A. Shaw, "Vowel recognition with time-delay neural network," *IEEE International Conference on Systems Engineering*, pp. 637–640, 1990.
- [9] S. S. Dhillon and S. Chakrabarti, "Power line interference removal from electrocardiogram using a simplified lattice based adaptive IIR notch filter," in *Engineering in Medicine and Biology Society, 2001. Proceedings of the 23rd Annual International Conference of the IEEE*, vol. 4, pp. 3407–3412, IEEE, 2001.
- [10] S. G. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 11, no. 7, pp. 674–693, 1989.